

# Minimizing Effective Many-Body Interactions

B. R. Barrett<sup>1)</sup>, D. C. Zheng<sup>1)</sup>, R. J. McCarthy<sup>2)</sup> and J. P. Vary<sup>3,4)</sup>

<sup>1)</sup> *Department of Physics, University of Arizona, Tucson, Arizona 85721*

<sup>2)</sup> *Department of Physics, Kent State University, Asatabula, Ohio 44004*

<sup>3)</sup> *Department of Physics and Astronomy, Iowa State University, Ames, IA 50011*

<sup>4)</sup> *Institute for Theoretical Physics, University of Heidelberg, Heidelberg, Germany*

## Abstract

A simple two-level model is developed and used to test the properties of effective interactions for performing nuclear structure calculations in truncated model spaces. It is shown that the effective many-body interactions sensitively depend on the choice of the single-particle basis and they appear to be minimized when a self-consistent Hartree-Fock basis is used.

# 1 Introduction

Assuming that only two-body interactions act among nucleons in nuclei, we can write the nuclear Hamiltonian as

$$H = \sum_{i=1}^A \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} . \quad (1)$$

This Hamiltonian can be divided into two terms,  $H_0$  and  $H_I$ , as

$$H = \sum_{i=1}^A \left( \frac{\mathbf{p}_i^2}{2m} + u_i \right) + \left( \sum_{i<j} v_{ij} - \sum_{i=1}^A u_i \right) \equiv H_0 + H_I, \quad (2)$$

where  $H_0$  is the one-body Hamiltonian which defines a single-particle basis and  $H_I$  is the two-body “residual” interaction. If the Schrödinger equation

$$H\Psi_i(1, 2, \dots, A) = E_i\Psi_i(1, 2, \dots, A) \quad (3)$$

could be solved in the infinite Hilbert space for a many-body system, the results would not depend on the choice of the one-body potential  $u$ , or, equivalently, the single-particle basis. In practice, in order to solve Eq.(3), one must truncate the infinite Hilbert space to a finite model space and introduce an effective interaction ( $V_{\text{eff}}$ ) to be used in the truncated model space. For an  $A$ -nucleon system, the effective interaction  $V_{\text{eff}}$  will, in principle, have two- [ $V_{\text{eff}}^{(2)}$ ], three- [ $V_{\text{eff}}^{(3)}$ ], ..., and  $A$ -body [ $V_{\text{eff}}^{(A)}$ ] parts. If the exact effective interaction, containing all (two- to  $A$ -body) components, can be obtained, the results will again be independent of the choice of the one-body potential  $u$ . However, three- and more-body effective interactions are difficult to calculate and are often ignored in practical shell-model calculations with the hope that they are small. One is then left with only the two-body effective interaction  $V_{\text{eff}}^{(2)}$  for performing nuclear structure calculations in a severely truncated model space.

With recent developments on the effective interaction theory, i.e., the use of the no-core model space [1, 2] and the Lee-Suzuki approach [3, 4, 5]

to the folded-diagram series, we can now calculate the effective two-body interaction accurately. The question is how to choose  $u$  to give the best approximation to the exact results when retaining only the  $V_{\text{eff}}^{(2)}$  part of the effective Hamiltonian.

In this work, we will use an exactly soluble two-level model (so that exact eigenenergies can be obtained) to show that the contribution to the ground-state energy from the often-neglected many-body effective interactions depend quite sensitively on the choice of the one-body potential  $u$ . We show that the Hartree-Fock (HF) self-consistent one-body field, appears to minimize the three- and more-body effects and consequently, the two-body effective interaction alone becomes a good approximation.

This paper is organized as follows. In the next section, we give a brief review of the effective interaction theories. In section 3, we introduce a simple two-level model and calculate the energy-independent two-body effective interaction for an arbitrarily chosen, non self-consistent, single-particle basis. In section 4, we repeat the calculation performed in section 3 for a self-consistent Hartree-Fock basis and compare the results with those obtained in section 3. Finally in section 5, we give our conclusions.

## 2 Energy-Independent Effective Interactions

When the full Hilbert space is divided into  $P$  and  $Q$  spaces with  $P$  projecting out the model space from the full Hilbert space and  $Q$  projecting out the excluded space, the  $P$ -space effective interaction for an eigenstate with an eigenenergy  $E$  can be written [6]

$$V_{\text{eff}}(E) = PH_IP + PH_IQ \frac{1}{E - QHQ} QH_IP, \quad (4)$$

Note that  $E$ , the energy of the many-body system which we wish to calculate, appears in the right-hand-side of the above equation. For this reason, the above effective interaction is also referred to as the energy-dependent

effective interaction.

About ten years ago, Lee and Suzuki [3] proposed a method for calculating an energy-independent effective interaction. This method is based on a “generalized” G matrix, which is defined as

$$G(\omega) = PH_IP + PH_IQ \frac{1}{\omega - QHQ} QH_IP . \quad (5)$$

This generalized G matrix, which depends on a starting energy  $\omega$ , is often referred to as the Q-box [7]. Obviously, the generalized G matrix, or the Q-box, becomes the energy-dependent effective interaction  $V_{\text{eff}}(E)$  when  $\omega$  is chosen to be equal to  $E$ .

It is easy to see that for a two-particle system ( $A=2$ ), the generalized G matrix becomes the Brueckner two-body G matrix [8]. Exact methods [9, 10] exist for calculating the Brueckner G matrix as a function of the starting energy  $\omega$ . However, the generalized G matrix is an  $A$ -body operator and is generally difficult to evaluate when  $A > 2$ . *If* it ever becomes possible to calculate  $G(\omega)$ , the *energy-independent* effective interaction  $V_{\text{eff}}$  can then be obtained through an iterative procedure proposed in Ref.[3]. Applying the Lee-Suzuki iterative procedure is equivalent to summing over all the folded diagrams which are not included in the generalized  $G$  matrix calculation. Further details on the Lee-Suzuki iterative methods can be found in Refs.[3, 4, 5].

In the case of a one-dimensional model space (one can always choose the model space to be one-dimensional no matter how many particles are involved), the effective interaction  $V_{\text{eff}}$  for the model space becomes a number (i.e., a one-by-one matrix). It is then apparent by comparing Eq.(4) and Eq.(5) that when the starting energy  $\omega$  happens to be one of the eigenenergies of the system,  $E$ , one has

$$V_{\text{eff}} = V_{\text{eff}}(E) = G(\omega = E) , \quad (6)$$

i.e., there is no need to utilize the iterative procedure and, therefore, the

contribution from the folded diagrams vanishes. In this case, no iterations are needed to obtain the effective interaction  $V_{\text{eff}}$  from  $G(\omega)$ .

In other words, in the case of a one-dimensional model space, the Lee-Suzuki approach to  $V_{\text{eff}}$  simplifies to the following equation for  $E$ :

$$E = PH_0P + G(E) , \quad (7)$$

which may be solved graphically. This has been shown explicitly in Ref.[5]. It is also shown in Ref.[5] that Eq.(7) is equivalent to the secular equation for the eigenenergies:

$$\det(H - EI) = 0 . \quad (8)$$

Therefore, the solutions to Eq.(7) correspond to the exact eigenenergies of the  $A$ -body system. Where it has been tested the method appears to work well. Of course, it can only produce eigenvalues for states having non-vanishing overlap with the model-space state.

Note that when the model space is  $D$ -dimensional ( $D > 1$ ), the energy-dependent effective interaction  $V_{\text{eff}}(E)$  is different from the the LS energy-independent effective interaction  $V_{\text{eff}}$ . Both the model space Hamiltonians  $[H_0 + V_{\text{eff}}(E)]$  and  $[H_0 + V_{\text{eff}}]$  have  $D$  eigenvalues. But for the former Hamiltonian, only one  $[E, \text{i.e., the one at which } V_{\text{eff}}(E) \text{ is evaluated}]$  out of  $D$  eigenvalues corresponds to a true eigenenergy of the system while, for the latter Hamiltonian, all its  $D$  eigenvalues are true eigenenergies of the system.

Because it is *not* known how to compute  $G(\omega)$  for a system of  $A$  nucleons, the usual procedure is to start with the nuclear (or Brueckner) two-body reaction matrix  $G^{(2)}$  [8] and try to calculate the two-body effective interaction  $V_{\text{eff}}^{(2)}$ . Assuming that the effective three- and higher-body forces are small, one can then use such an effective two-body interaction as input to shell-model programs, such as the OXBASH code [11] to perform model-space diagonalizations for the entire system. We briefly outline how one obtains the effective two-body interaction  $V_{\text{eff}}^{(2)}$  from the starting-energy-dependent Brueckner two-body  $G$  matrix.

The Brueckner  $G$  matrix represents the infinite summation (ladder sum) of two-particle scatterings. This is defined, in analogy to Eq.(5) for  $G(\omega)$ , as

$$G^{(2)}(\omega_2) = P_2 H_I P_2 + P_2 H_I Q_2 \frac{1}{\omega_2 - Q_2 H Q_2} Q_2 H_I P_2, \quad (9)$$

where  $P_2$  and  $Q_2$  are now two-particle projection operators, which determine the allowed and forbidden intermediate states into which the two particles can scatter. Note that  $G^{(2)}$  is defined to depend on a starting energy  $\omega_2$ . It is clear that this starting energy refers to a two-particle system and when one approximates  $V_{\text{eff}}$  by omitting many-body interactions,  $\omega_2$  is different from the starting energy  $\omega$  in  $G(\omega)$ .

The  $\omega_2$ -independent two-body effective interaction  $V_{\text{eff}}^{(2)}$  can be obtained from  $G^{(2)}(\omega_2)$  by summing over all the folded diagrams with two valence lines. This can now be accomplished without much difficulty by applying the Lee-Suzuki iterative procedure [3, 4, 5].

### 3 $V_{\text{eff}}^{(2)}$ for an Arbitrary Basis

A simple two-level model, consisting of two single-particle states  $|1\rangle$  and  $|2\rangle$ , is used for our investigation. Each level can hold up to four nucleons: spin-up proton ( $p \uparrow$ ), spin-down proton ( $p \downarrow$ ) spin-up neutron ( $n \uparrow$ ) and spin-down neutron ( $n \downarrow$ ). The Hamiltonian for this model is determined by  $H_0$ , represented by the single-particle energies (SPE) of the two levels ( $\epsilon_1$  and  $\epsilon_2$ ), and  $H_I$ , represented by 14 antisymmetrized, normalized two-body matrix elements (TBME)  $[\langle ab|H_I|cd\rangle_{JT}]$ . Our choice for the SPE and the TBME is given in Table 1.

As our first calculation, we diagonalize, using the OXBASH shell-model code [11], the Hamiltonian in the *full* two-level space for  $A=2, 3$  and 4. The results obtained are exact and are given in Table 2 in the column under the heading “Exact”.

Table 1: The one-body part  $H_0$ , defined by the SPE  $\epsilon_1$  and  $\epsilon_2$ , and the two-body part  $H_I$ , defined by the antisymmetrized, normalized TBME  $\langle ab|H_I|cd\rangle_{J,T}$ , of the Hamiltonian  $H$ . For later convenience, we also denote each matrix element by  $a_{ij}$ ,  $b_{ij}$ , *etc.*.

$\epsilon_1 = 0,$	$\epsilon_2 = 10$
$a_{11} = \langle 11 H_I 11\rangle_{0,1} = -3.0$	$b_{11} = \langle 11 H_I 11\rangle_{1,0} = -4.0$
$a_{12} = \langle 11 H_I 12\rangle_{0,1} = -1.5$	$b_{12} = \langle 11 H_I 12\rangle_{1,0} = -2.0$
$a_{13} = \langle 11 H_I 22\rangle_{0,1} = -1.7$	$b_{13} = \langle 11 H_I 22\rangle_{1,0} = -1.9$
$a_{22} = \langle 12 H_I 12\rangle_{0,1} = -2.7$	$b_{22} = \langle 12 H_I 12\rangle_{1,0} = -2.9$
$a_{23} = \langle 12 H_I 22\rangle_{0,1} = -1.6$	$b_{23} = \langle 12 H_I 22\rangle_{1,0} = -2.1$
$a_{33} = \langle 22 H_I 22\rangle_{0,1} = -2.5$	$b_{33} = \langle 22 H_I 22\rangle_{1,0} = -2.9$
$c_{11} = \langle 12 H_I 12\rangle_{0,0} = 0.0$	$d_{11} = \langle 12 H_I 12\rangle_{1,1} = 0.0$

Next we truncate the two-level space to a smaller space containing only the lower level, which means that we must construct the effective Hamiltonian appropriate for this model space.

As we mentioned in the Introduction, for the truncated space, the effective interaction  $V_{\text{eff}}$  for an  $A$ -body system can be written as

$$V_{\text{eff}} = V_{\text{eff}}^{(2)} + V_{\text{eff}}^{(3)} + \cdots + V_{\text{eff}}^{(A)}. \quad (10)$$

When only the two-body part  $V_{\text{eff}}^{(2)}$  is kept in shell-model calculations, the choice of the single-particle basis becomes important as we shall see below. In this section, we will use the original single-particle basis ( $|1\rangle$  and  $|2\rangle$ , as used in Table 1), which is not a self-consistent basis, to calculate the effective two-body interaction  $V_{\text{eff}}^{(2)}$ . In the next section, we will introduce a self-consistent HF basis for  $A=4$  and re-calculate  $V_{\text{eff}}^{(2)}$  and compare the results obtained in these two sections.

To obtain the energy-independent effective two-body interaction  $V_{\text{eff}}^{(2)}$ , we start with the starting energy-dependent two-body G matrix and apply

Table 2: The  $A=2, 3$ , and 4 ground-state energies in the two-level model from *exact* matrix diagonalizations and from the one-level model space calculations with a two-body effective Hamiltonian obtained in the arbitrary and in the self-consistent HF bases.

Ground-state energy	Exact	Arbitrary Basis	Self-Consistent Basis
$E_2(J=0, T=1)$	-3.390	-3.390	-3.390
$E_2(J=1, T=0)$	-4.582	-4.582	-4.582
$E_3(J=1/2, T=1/2)$	-12.938	-11.958	
$E_4(J=0, T=0)$	-27.703	-23.916	-27.645

the Lee-Suzuki procedure [3]. Previously we labelled this as  $G^{(2)}$  but now we relabel it as  $G^{JT}$  to indicate the two-body conserved quantities. For the Hamiltonian and single-particle basis defined in Table 1, the  $G^{JT}$  matrix element for the  $J=0, T=1$  channel for the chosen model space is given by

$$\begin{aligned}
G^{01}(\omega_2) &= a_{11} + \begin{pmatrix} a_{12}, & a_{13} \end{pmatrix} \begin{pmatrix} \omega_2 - (\epsilon_1 + \epsilon_2 + a_{22}) & -a_{23} \\ -a_{32} & \omega_2 - (2\epsilon_2 + a_{33}) \end{pmatrix}^{-1} \begin{pmatrix} a_{21} \\ a_{31} \end{pmatrix} \\
&= (-3.0) + \begin{pmatrix} -1.5, & -1.7 \end{pmatrix} \begin{pmatrix} \omega_2 - 7.3 & 1.6 \\ 1.6 & \omega_2 - 17.5 \end{pmatrix}^{-1} \begin{pmatrix} -1.5 \\ -1.7 \end{pmatrix}. \quad (11)
\end{aligned}$$

Similarly, for the  $J=1, T=0$  channel, we have

$$G^{10}(\omega_2) = (-4.0) + \begin{pmatrix} -2.0, & -1.9 \end{pmatrix} \begin{pmatrix} \omega_2 - 7.1 & 2.1 \\ 2.1 & \omega_2 - 17.1 \end{pmatrix}^{-1} \begin{pmatrix} -2.0 \\ -1.9 \end{pmatrix}. \quad (12)$$

Clearly, these two-particle  $G$  matrix elements depend on the starting energy  $\omega_2$ . As in Ref.[4, 5], we now use the Lee-Suzuki method [3] to obtain the energy-independent two-body effective interaction, which is equivalent to summing over the two-particle folded diagrams to all orders. We obtain



the following results:

$$(V_{\text{eff}}^{(2)})^{01} = \begin{cases} -3.390 & \text{if } \omega_2 < 1.976 , \\ 7.342 & \text{if } 1.976 < \omega_2 < 12.595 , \\ 17.848 & \text{if } \omega_2 > 12.595 , \end{cases} \quad (13)$$

and

$$(V_{\text{eff}}^{(2)})^{10} = \begin{cases} -4.582 & \text{if } \omega_2 < 1.288 , \\ 7.157 & \text{if } 1.288 < \omega_2 < 12.391 , \\ 17.625 & \text{if } \omega_2 > 12.391 . \end{cases} \quad (14)$$

It is easy to verify that the above numbers are precisely the eigenenergies of the two-particle system ( $A=2$ ) whose Hamiltonian is defined by Table 1, as they must be [5], since it is just an application of Eqs.(6) and (7) to a simple  $A=2$  case.

The ground-state energies of the  $A=3$  and  $A=4$  systems, when we neglect effective many-body terms in Eq.(10), can be expressed in terms of these two matrix elements as

$$E_3 = 3\epsilon_1 + 1.5 \left[ (V_{\text{eff}}^{(2)})^{01} + (V_{\text{eff}}^{(2)})^{10} \right] , \quad (15)$$

and

$$E_4 = 4\epsilon_1 + 3.0 \left[ (V_{\text{eff}}^{(2)})^{01} + (V_{\text{eff}}^{(2)})^{10} \right] . \quad (16)$$

The results are (noting that  $\epsilon_1=0$ ):

$$E_3 = 1.5(-3.390 - 4.582) = -11.958 , \quad (17)$$

and

$$E_4 = 3.0(-3.390 - 4.582) = -23.916 . \quad (18)$$

These differ from the *exact* results of  $-12.938$  for  $E_3$  and  $-27.703$  for  $E_4$  by about 1.0 and 3.8, respectively. The discrepancies reflect the importance of the neglected effective three- and (for  $A=4$ ) four-body terms.

## 4 $V_{\text{eff}}^{(2)}$ for a Self-Consistent Basis

Addressing the case with the largest discrepancy, we now consider a self-consistent HF single-particle basis for the ground state of the  $A=4$  system. We rewrite the original Hamiltonian of Table 1 as

$$H = (H_0 + U) + (H_I - U) \equiv H'_0 + H'_I \quad (19)$$

In the previous section, we have set  $U=0$ . In this section, we use a self-consistent one-body field  $U$  generated by all the  $A$  particles in the system. It can be obtained iteratively by using the following equations where  $n$  represents the iteration number:

$$\langle \alpha_n | U_n | \beta_n \rangle = \sum_{J,T,\gamma=\text{occ.}} \frac{(2J+1)(2T+1)}{2(2j_\alpha+1)} \sqrt{(1+\delta_{\alpha_{n-1},\gamma_{n-1}})(1+\delta_{\beta_{n-1},\gamma_{n-1}})} \langle \alpha_{n-1}\gamma_{n-1} | H_I | \beta_{n-1}\gamma_{n-1} \rangle_{J,T}, \quad (20)$$

$$(H_0 + U_n) | \alpha_n \rangle = \epsilon_{\alpha_n} | \alpha_n \rangle, \quad (21)$$

where  $|\alpha_i\rangle$ ,  $|\beta_i\rangle$ ,  $|\gamma_i\rangle$ , *etc.* are the single-particle states in the  $i$ -th iteration and the summation ( $\gamma$ ) is over the occupied states. The TBME in the above expression are normalized and antisymmetrized. When converged, we obtain a self-consistent HF basis.

It is obvious from the above equations that the resulting single-particle basis, defined by  $H'_0 = (H_0 + U)$ , and the two-body residual interaction  $H'_I = (H_I - U)$  are mass-dependent and should only be applied to the system for which they are calculated.

It is easy to work out the self-consistent HF basis for  $A=4$ , which is a “closed-shell” system in our two-level model. The new single-particle states, denoted by  $|1'\rangle$  and  $|2'\rangle$ , are linear combinations of the old ones:

$$|1'\rangle = 0.92785|1\rangle + 0.37295|2\rangle, \quad (22)$$

$$|2'\rangle = -0.37295|1\rangle + 0.92785|2\rangle. \quad (23)$$

The corresponding single-particle energies are:

$$H'_0|1'\rangle = -15.0323|1'\rangle, \quad (24)$$

$$H'_0|2'\rangle = 5.3808|2'\rangle. \quad (25)$$

The TBME of the residual interaction ( $H'_I = H_I - U$ ), evaluated using the new basis, are given in Table 3.

Note that the TBME of  $H'_I$  listed in Table 3 are  $A$ -dependent not only because the one-body potential  $U$  is  $A$ -dependent but also due to the fact that when TBME are calculated for a one-body potential, a factor  $1/(A-1)$  has to be introduced:

$$U = \sum_{i=1}^A u_i = \frac{1}{A-1} \sum_{i<j}^A (u_i + u_j). \quad (26)$$

With the self-consistent single-particle basis, there is no coupling between the  $0p\text{-}0h$  configuration and the  $1p\text{-}1h$  configuration. This is guaranteed by the following equation:

$$\sum_{J,T} (2J+1)(2T+1) \langle 1'1' | H'_I | 1'2' \rangle_{J,T} = 0. \quad (27)$$

Now in the HF basis, we can obtain an estimate of the ground-state energy of  $A=4$  even before calculating  $G^{JT}$  by using the lowest-order in  $H'_I$  estimate of  $V_{\text{eff}}^{(2)}$  in Eq.(16):

$$E_4^{\text{HF}} = 4\epsilon_{1'} + 3 (\langle 1'1' | H'_I | 1'1' \rangle_{0,1} + \langle 1'1' | H'_I | 1'1' \rangle_{1,0}), \quad (28)$$

which gives  $E_4^{\text{HF}} = -27.283$  MeV. Note that this result is already closer to the exact energy of  $-27.703$  MeV than the result obtained in the previous section for  $A=4$ .

For the HF basis, we then calculate the effective two-body interaction for the truncated model space. The results are

$$(V_{\text{eff}}^{(2)})^{01} = 6.091, \quad (V_{\text{eff}}^{(2)})^{10} = 4.738. \quad (29)$$

Table 3: The single-particle energies of  $H'_0 = (H_0 + U)$  and the two-body matrix elements of  $H'_I = (H_I - U)$  for the Hamiltonian defined in Table 1 with the self-consistent HF single-particle basis:  $|1'\rangle = 0.92785|1\rangle + 0.37295|2\rangle$ , and  $|2'\rangle = -0.37295|1\rangle + 0.92785|2\rangle$ .

$\epsilon_{1'} = -15.0323,$	$\epsilon_{2'} = 5.3808$
$\langle 1'1' H'_I 1'1'\rangle_{0,1} = 6.1415$	$\langle 1'1' H'_I 1'1'\rangle_{1,0} = 4.8074$
$\langle 1'1' H'_I 1'2'\rangle_{0,1} = 0.0541$	$\langle 1'1' H'_I 1'2'\rangle_{1,0} = -0.0541$
$\langle 1'1' H'_I 2'2'\rangle_{0,1} = -1.3402$	$\langle 1'1' H'_I 2'2'\rangle_{1,0} = -1.6120$
$\langle 1'2' H'_I 1'2'\rangle_{0,1} = 4.5702$	$\langle 1'2' H'_I 1'2'\rangle_{1,0} = 4.2265$
$\langle 1'2' H'_I 2'2'\rangle_{0,1} = 1.2156$	$\langle 1'2' H'_I 2'2'\rangle_{1,0} = 0.8955$
$\langle 2'2' H'_I 2'2'\rangle_{0,1} = 0.7399$	$\langle 2'2' H'_I 2'2'\rangle_{1,0} = 0.8176$
$\langle 1'2' H'_I 1'2'\rangle_{0,0} = 6.5505$	$\langle 1'2' H'_I 1'2'\rangle_{1,1} = 6.5505$

So the ground-state energy for  $A=4$ , using an effective two-body interaction in a self-consistent basis, is

$$E_4 = 4\epsilon_{1'} + 3.0 \left[ (V_{\text{eff}}^{(2)})^{01} + (V_{\text{eff}}^{(2)})^{10} \right] = -27.645. \quad (30)$$

This is extremely close to the exact result of -27.703 MeV.

It should be pointed out that the good agreement with the exact result is mainly due to the fact that, with the HF basis, the matrix elements  $\langle 1'1'|H'_I|1'2'\rangle_{0,1}$  and  $\langle 1'1'|H'_I|1'2'\rangle_{1,0}$  are very small (see Table 3). As we have mentioned previously, the sum of these two matrix elements vanishes in the HF basis [Eq.(27)]. We further notice that the exact ground-state energy (i.e., the energy obtained in the full-space matrix diagonalization) of the  $A=4$  system,  $E_4$ , does not depend the magnitude of each of the two matrix elements, which we denote by  $x$ :  $x = |\langle 1'1'|H'_I|1'2'\rangle_{0,1}| = |\langle 1'1'|H'_I|1'2'\rangle_{1,0}|$ . However, it is clear that the effective 2-, 3-, and 4-body contributions to  $E_4$  do depend on  $x$ . In fact, as shown in Fig.1, the leading-order diagrams for the effective 2-, 3-, and 4-body forces involve  $x$ . Therefore, when we

increase  $x$ , the effective 2-, 3-, and 4-body interactions will all change but these changes produce no net effect on  $E_4$  which remains the same. In particular, one can show that as  $x$  increases, the effective 2-body interaction becomes more attractive while the effective 3-body interaction becomes more repulsive. The smallness of  $x$  in the HF basis means that the effective many-body interactions are minimized and, consequently, the result using only the effective two-body interaction is in good agreement with the exact result.

## 5 Conclusions

With the Lee-Suzuki iterative method, it is now feasible to calculate the exact, energy-independent, effective two-body interaction for a no-inert-core model space from the starting-energy-dependent Bruckner G matrix  $G^{(2)}(\omega)$ . We have demonstrated through a simple two-level model that when only the two-body effective interaction is used for shell-model calculations in a truncated model space, the choice of the single-particle basis is very important. An optimal choice of the single-particle basis should be chosen to minimize the neglected three- and more-body effective interactions. We have shown that a self-consistent HF basis serves this purpose very well for the two-level model. Encouraged by these results, it is now worth examining the more realistic situation when one must calculate  $G^{(2)}(\omega_2)$  in some chosen basis *before* any self-consistency calculation may be attempted. The present results indicate that one should choose a realistic single-particle  $H_0$  for each  $A$  (assuming realistic is close to self-consistent) in order to minimize the neglected effective many-body forces.

## Acknowledgments

We thank the Department of Energy for partial support during our stay at the Institute for Nuclear Theory at the University of Washington, where

this work was initiated. Two of us (B.R.B. and D.C.Z.) also acknowledge partial support of this work by the National Science Foundation, Grant No. PHY91-03011. One of us (J.P.V.) acknowledges partial support by the U.S. Department of Energy under Grant No. DE-FG02-87ER-40371, Division of High Energy and Nuclear Physics and partial support from the Alexander von Humboldt Foundation.

## References

- [1] J.P. Vary, in *Theory and Applications of Moment Methods in Many-Fermion Systems*, eds B.J. Dalton, S.M. Grimes, J.P. Vary, and S.A. Williams, Plenum Press (New York), 1980, p.423;
- [2] L. Jaqua, P. Halse, B.R. Barrett, and J.P. Vary, “Effective shell-model interaction for no-core model space”, University of Arizona preprint, AZPH-TH/93-08.
- [3] S.Y. Lee and K. Suzuki, Phys. Lett. **91B**, 79(1980); K. Suzuki and S.Y. Lee, Prog. of Theor. Phys. **64**, 2091(1980).
- [4] N.A.F.M. Poppelier and P.J. Brussaard, Nucl. Phys. **A530**, 1(1991).
- [5] D.C. Zheng, J.P. Vary and B.R. Barrett, *Exact Solutions of Model Hamiltonian Problems with Effective Interactions*, University of Arizona preprint, AZPH-TH/92-40 (to be published in Nuclear Physics A).
- [6] H. Feshbach, Ann. Phys. (N.Y.) **19**, 287(1962).
- [7] T.T.S. Kuo, in *Lecture Notes in Physics*, **144**, eds T.T.S. Kuo and S.S.M. Wong (Springer, Berlin, 1981), p.248.
- [8] K.A. Brueckner, Phys. Rev. **97**, 1353(1955); **100**, 36(1955).
- [9] B.R. Barrett, R.G.L. Hewitt, and R.J. McCarthy, Phys. Rev. **C 3**, 1137(1971).
- [10] J.P. Vary and S.N. Yang, Phys. Rev. **C 15**, 1545(1977).

- [11] A. Etchegoyen, W.D.M. Rae, N.S. Godwin, B.A. Brown, W.E. Ormand, and J.S. Winfield, the Oxford–Buenos Aires — MSU Shell Model Code (OXBASH) (unpublished).

## Figure Caption

**Fig.1** The leading-order diagrams for the effective 2-, 3-, and 4-body forces.